6.9 An incompressible viscous fluid is placed between two large parallel plates as shown. The bottom plate is fixed and the upper plate moves with a constant velocity, \( U \). For these conditions the velocity distribution between the plates is linear and can be expressed as \( u = \frac{U}{b} \). Determine: (a) the volumetric dilatation rate. (b) the rotation vectors. (c) the vorticity, and (d) the rate of angular deformation.

(a) Volumetric dilatation rate \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \)

(b) Rotation vectors \( \vec{\omega} = \omega \hat{z} \)

\[
\frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{z} = -\frac{U}{2b} \hat{z}
\]

(c) Vorticity \( \vec{\gamma} = 2\vec{\omega} = -\frac{U}{b} \hat{z} \) (Vorticity \( \vec{\gamma} \) 's vortex)

(d) Rate of angular deformation \( \dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{U}{b} \) (force)

6.37 It is known that the velocity distribution for two-dimensional flow of a viscous fluid between wide parallel plates as shown is parabolic; that is, \( u = U_c \left[ 1 - \left( \frac{y}{h} \right)^2 \right] \)

With \( v = 0 \). Determine, if possible, the corresponding stream function and velocity potential.

(a) Stream function?

Stream function \( \psi \) 与速度 U, V 的关系

\[
\begin{align*}
U &= \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x} \\
\end{align*}
\]

\( \int_{x_0}^x u = \frac{\partial \psi}{\partial y} = U_c \left[ 1 - \left( \frac{y}{h} \right)^2 \right] \to \text{Jordan (对 y 贡献)} \)

\( \int d\psi = \int U_c \left[ 1 - \left( \frac{y}{h} \right)^2 \right] dy \to \psi = U_c \left[ y - \frac{y^3}{3h^2} \right] + f_i(x), \quad f_i(x) = ? \)

利用 \( v = -\frac{\partial \psi}{\partial x} = f_i'(x) = 0 \), \( \frac{\partial \psi}{\partial x} \) constant C.

因此，Stream function \( \psi = U_c \left[ y - \frac{y^3}{3h^2} \right] + C \).

(b) Velocity potential?

Velocity potential \( \phi \) 与速度 U, V 的关系

\[
\begin{align*}
U &= \frac{\partial \phi}{\partial x}, \quad V = -\frac{\partial \phi}{\partial y} \\
\end{align*}
\]

\( \int \phi = \int U_c \left[ 1 - \left( \frac{y}{h} \right)^2 \right] dx \to \phi = U_c \left[ 1 - \left( \frac{y}{h} \right)^2 \right] \phi_{i} \), \( \phi_{i} \) = ?

利用 \( V = \frac{\partial \phi}{\partial y} = -2U_c \frac{\partial \phi}{\partial x} = f_i(y) = 0 \) 可知没有 \( x, y \) 可知 \( V = 0 \)

因此，该流动可以用来描述此流动，因此流动是非 irrotational flow.
6.47 It is suggested that the velocity potential for the incompressible, nonviscous, two-dimensional flow along the wall as shown is \( \phi = r^{4/3} \cos \frac{4}{3} \theta \). Is this a suitable velocity potential for flow along the wall? Explain. If the potential \( \phi \) is suitable, then the wall's stream function \( \psi \) is

\[
\psi = \frac{1}{r} \cdot \frac{\partial \phi}{\partial \theta} = \frac{4}{3} r^{1/3} \cos \frac{4}{3} \theta - c \Rightarrow \text{boundary condition}
\]

\[
\int d\psi = \int \frac{4}{3} r^{1/3} \cos \frac{4}{3} \theta d\theta \Rightarrow \psi = r^{4/3} \sin \frac{4}{3} \theta + f_2(\theta) - c
\]

\[
\int d\psi = \int \frac{4}{3} r^{1/3} \sin \frac{4}{3} \theta dr \Rightarrow \psi = r^{4/3} \sin \frac{4}{3} \theta + f_2(\theta) - c
\]

satisfies (1), (2) \( \psi = r^{4/3} \sin \frac{4}{3} \theta + C \); \( \frac{\partial \psi}{\partial n} \) wall, \( \theta = 0, \); \( \theta = \frac{3}{4} \pi \), \( \psi = \text{constant} \)

Therefore, the velocity potential is suitable.

6.95 A viscous fluid (specific weight = 1.26 kN/m\(^3\); viscosity = 1.4 N·s/m\(^2\)) is contained between two infinite, horizontal parallel plates as shown. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity \( U \) while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.25 cm. If the upper plate moves with a velocity of \( 6 \times 10^{-3} \) m/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.

Equation 6.12 : \( \nabla = 0 \Rightarrow \frac{\partial ^2 \psi}{\partial y^2} + \frac{\partial ^2 \psi}{\partial x^2} = 0 \) (Continuity eq.)

\[
\Rightarrow u = u(y) \Rightarrow \frac{\partial \psi}{\partial y} = c \text{, } y + c_1, \text{ } c_1 = ?, \text{ } c_2 = ?
\]

Boundary condition : \( y = 0, \text{ } u = 0 \); \( y = b, \text{ } u = U \)

Velocity distribution : \( u = \frac{y}{b} + \frac{1}{2\mu} \left( \frac{\partial \phi}{\partial y} \right) (y - b) \)

Max. velocity : \( \frac{\partial ^2 \psi}{\partial y^2} = \frac{\partial \psi}{\partial x} = 0 \Rightarrow \frac{\partial \psi}{\partial y} = \frac{U}{b} + \frac{1}{2\mu} \left( \frac{\partial \phi}{\partial x} \right) (2y - b) \)

\[
\Rightarrow \frac{\partial \psi}{\partial y} = \frac{U}{b} + \frac{1}{2\mu} \left( \frac{\partial \phi}{\partial x} \right) \Rightarrow \frac{\partial \psi}{\partial x} = \frac{U}{b} \frac{\partial \phi}{\partial x}
\]

Manometer : \( \psi = \frac{\partial \phi}{\partial x} \text{, } x = \frac{\phi_{h1} - \phi_{h2}}{b} \Rightarrow \text{constant} = 7.75 \text{ kN/m}^2 \)

\[
\frac{\partial \phi}{\partial x} = \frac{P_1 - P_2}{\mu \frac{d \psi}{d x}} = \frac{7.75 \text{ kN/m}^2}{6.15 \text{ m}} = -51.667 \text{ kN/m}^2
\]

\[
\psi = \left( 1.4 \text{ N·s/m} \right) \left( 6 \times 10^{-3} \text{ m/s} \right) \text{, } \frac{0.025 \text{ m}}{2} = 1.9 \text{ cm}
\]
6.54 水从一个平坦的表面以 1.2 m/s 的速度流动。通过一个窄缝的水泵抽取水，抽水速度为 0.01 m³/s 每米缝长。假设流体是不可压缩的和无粘性的，可以用均匀流和源的组合来表达。找到边界上的停留点（点 A），并确定停留流线的方程。如果流体必须位于距离表面 H 多远的地方才能不被吸入窄缝？

\[ V_r = U \sin \theta - \frac{m}{2\pi} \theta \]

\[ V_t = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad V_\theta = -\frac{\partial \psi}{\partial r} \]

均匀流和源流组合的流场如何描述？速度？

\[ V_r = U \cos \theta - \frac{m}{2\pi r}, \quad V_\theta = 0 \]

在边界上，\( \theta = 0, \pi \)，流体为零。停留点的方程为 \( \psi = \frac{m}{2\pi U} \)。

停留点的流量为 \( \frac{m}{2\pi U} \)。停留点距离为 \( \frac{m}{2\pi U} \)。停留点的流线方程为 \( U \sin \theta - \frac{m}{2\pi} \theta = 0 \)

令 \( \sin \theta = y \)，则流线方程可写为 \( y = \frac{m}{2\pi U} \theta \)

该方程可视作 Solid boundary，上方的流体不会穿越，流入窄缝中。

距离 Wall 多远？最大距离 \( H \) 发生在 \( \theta = \pi \) 处，

\[ H = \frac{m}{2\pi U} \cdot \pi = \frac{m}{2U} = \frac{0.02 \text{m}^3/\text{s}}{2 \times 1.2 \text{m/s}} = 8.33 \times 10^{-3} \text{m} \]